Hahn fields, automorphisms and linear recurrence

Michele Serra and Lothar Sebastian Krapp
(joint work with Salma Kuhlmann)

Abstract

Let k be a field, G a totally ordered abelian group. The maximal field of generalised power series $\mathbb{K} = k((G))$ consists of all formal expressions of the form $a = \sum_{g \in G} a_g t^g$ where $a_g \in k$ and the support $\operatorname{supp}(a) = \{g \in G : a_g \neq 0\}$ is well-ordered in G. Most prominently, $k((\mathbb{Z}))$ consists of all Laurent series with coefficients in k. The field $\mathbb{K}$ can be endowed with a canonical valuation v and, also due to Kaplansky's Embedding Theorem, plays a fundamental role in the classification of valued fields. Consider a field K such that $k(G) \subseteq K \subseteq \mathbb{K}$, where $k(G)$ is the fraction field of the group ring $k[G] = \{s \in \mathbb{K} : \operatorname{supp}(a) \text{ is finite}\}$. We call K a Hahn field.

In the first part of this talk we introduce Hahn fields, illustrate their main features and present some results about the group $v\text{-Aut}\, K$ of valuation preserving automorphisms of a Hahn field K. These will include, under certain conditions, decomposing $v\text{-Aut}\, K$ into a 4-factor semi-direct product of notable subgroups.

Within fields of Laurent series, those series representing rational functions can be characterised via linear recurrence relations of their coefficients. Inspired by this result, we introduce in the second part of the talk the notion of *generalised* linear recurrence relations within fields of generalised power series. Such relations determine distinguished Hahn fields in $\mathbb{K}$, covering Rayner fields, and also relate to certain lifting properties, which are of particular interest in the study of $v\text{-Aut}\, K$.

References

[1] L. S. Krapp, S. Kuhlmann, M. Serra, On Rayner Structures, Comm. Algebra 50 (2022) 940–948, DOI:10.1080/00927872.2021.1976789.

[2] L. S. Krapp, S. Kuhlmann, M. Serra, Generalised power series determined by linear recurrence relations, Preprint, https://doi.org/10.48550/arXiv.2206.04126.

[3] S. Kuhlmann & M. Serra, The automorphism group of a valued field of generalised formal power series, J. Algebra, Vol. 605, 2022, Pages 339–376, DOI:10.1016/j.jalgebra.2022.04.023.

[4] M. Serra, Automorphism groups of hahn groups and hahn fields, Ph.D. thesis, Universität Konstanz (2021).
http://nbn-resolving.de/urn:nbn:de:bsz:352-2-19yqyhucnbtlw9